Nuclear Structure Study of Some Actinide Nuclei

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A modified version of the previously proposed exponential model with pairing attenuation for the well deformed even-even nuclei has been applied to predict the energy levels of doubly even actinide nuclei. Satisfactory results are obtained by that model as compared with the experimental results. The backbending phenomena are successfully described and discussed. A further comparison with the main previous models has been undertaken to confirm its validity in the heavy nuclei region.

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I. INTRODUCTION

Theoretical predictions of the existence of superheavy nuclei are based on the calculated properties of heavy nuclei as a function of the location of the single particle orbits within deformed potentials[1, 2]. The transuranic nuclei furnish a testing ground for these theories where the calculated properties may be compared with experiment.

Accurate experimental data in the actinide nuclei mass region should result in improved nuclear models including the microscopic level structure. The states of even–even nuclei in this region appear not far below the energy gap defined by the odd–even mass difference, and an interplay between single particle and collective aspects may be expected.

Many models have been applied to discuss the collective states of even – even nuclei in the actinide region [3, 4, 5, 6, 7, 8, 9, 10]. At low angular momentum, the yrast band of deformed nuclei can be reasonably described up to the point where there has been any band crossing [11]. The breakup of a pair of nucleus leads to band crossing and makes up a backbending. This has been implemented in the two-quasiparticle decoupling model [12] and the cranking shell model [13] and has gained considerable success. This mechanism has been also incorporated into the interacting boson model [6, 14, 15]. Furthermore, a model of rotator based in the q-poincare energy mass difference, have been applied to describe the energy level of some even–even nuclei and rather successful results have been obtained before the backbending region [16].

The main purpose of the present work is to investigate Th, U and Pu even–even nuclei in a phenomenological way in the framework of a three parameters formula [17] based on the exponential model with pairing attenuation [18]. It is hoped by such work to have a good description of the backbending regions besides those of the low lying states of some actinide nuclei. A comparison with the main previus models revels its validity in actinid region

II. NUCLEAR MODELS DESCRIPTION

The energy levels of the ground state bauds in even–even nuclei can be interpreted on the basis of a semiclassical model, in which the energy contains in addition to the usual rotational term, a potential energy term which depends on the difference of the moment of inertia φ_I (for the state of angular momentum I) from that of the ground state φ_0 [5]. This model is called the variable moment of inertia model (VMI). In this model they assumed that there exists a variational expression for the energy in the form:

$$E_I = \frac{I\left(I+1\right)}{2\varphi_I} + \frac{1}{2}C\left(\varphi_I - \varphi_0\right)^2 \tag{1}$$

where C is the restoring force constant.

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A nuclear softness (NS) model for the energy levels of the ground state bands in even–even nuclei is proposed by treating the variation of the moment of inertia with I in a more generalized manner [2] and the level energy is given by

$$E(I) = \frac{AI(I+1)}{(1+\delta_1 I + \delta_2 I^2)} \tag{2}$$

The results of the calculation of this three parameter expression are denoted as NS(3).

The interacting boson model (IBM) assumes that an even – even nucleus consists of an inert core plus some valence particles, which are those outside the closed shells at 28, 50, 82 and 126 tend to pair together in states with angular momentum L= 0 and 2 and called S-bosons and d-bosons respectively, these pairs are treated as bosons. If no distinction is made between proton and neutron bosons this referred to as interaction boson model-1 or IBM-1. The interacting boson model–2 or IBM-2 distinguishes between protons and neutron bosons [6] There are three possible analytic solutions which are SU(5), SU(3) and SU(6), their corresponding energy formulas are well known [6].

An exponential model with pairing attenuation was developed by Sood and Jain [18] based on the exponential dependence of the nuclear moment of inertia on pairing correlation [4] and on an explicit spin dependence of the pairing gap parameter. They gave the following rotational energy expression:

$$E(I) = \frac{\hbar^2}{2\varphi_0} I(I+1)\, Exp[\Delta_0 \left(1 - \frac{I}{I_c}\right)^{\frac{1}{2}} \tag{3}$$

This approach was used to fit the energies of the yrast band levels in well-deformed nuclei and excellent satisfactory results were obtained up to the point where backbending occurs. The forward or the down–bending region in $\varphi - \omega^2$ plots is not included in their calculations.

A recent modification of the exponential model with paining attenuation (Eq. 3) has been undertaken by letting ν to be a free parameter in the following expression[17]:

$$E(I) = \frac{\hbar^2}{2\varphi_0} I(I+1)\, Exp\left[\Delta_0 \left(1 - \frac{I}{I_c}\right)^{\frac{1}{\nu}}\right] \tag{4}$$

where φ_0, Δ_0 and ν are the three parameters of the model which are adjusted to give a least square fit to the experimental data for low and high angular momenta. Also, I_c corresponds to the minimum value of the root mean square deviation.For all the even-even nuclei considered in this paper, the experimental data are taken from [19] and recent Nuclear Data Dheets.

III. BACKBENDING PHENOMENA IN SOME EVEN-EVEN ACTINIDE NUCLEI (TH, U, PU)

In the present work an attempt is made to study the behavior of the backbending phenomena in the ground state bands for Th, U and Pu even–even nuclei using the VMI, NS3 models along with the present improved modified exponential model with pairing attenuation. The interacting boson model predictions are excluded because of their complexity (eight parameters in some cases) [14]. Furthermore, the well known energy expressions (SU(5), SU(3) and SU(6) are far from being suitable to describe the backbends in the $\varphi - \omega^2$ plots.

The plots of the calculated data of $2\varphi_I/\hbar^2$ versus $(\hbar\omega)^2$ for the aforementioned isotopes are given in Fig. 1, where the experimental data are also presented. From the excitation energies $E(I)$ of the ground state bands we deduce the nuclear moment of irentia and the squared rotational frequency ω^2 by using the most sensitive relations:

$$\frac{2\varphi}{\hbar^2} = \frac{4I-2}{E(I) - E(I-2)}\ , \tag{5}$$

and

$$(\hbar\omega)^2 = \left(I^2 - I + 1\right)\left[\frac{E(I) - E(I-2)}{2I-1}\right]^2 \tag{6}$$

TABLE I: Experimental and calculated energy (in MeV) levels of the ground state bands of even-even actinide nuclei.

I		2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+	18^+	20^+	22^+	24^+	26^+	28^+	30^+
^{224}Th	Exp.	0.0981	0.2841	0.5347	0.8339	1.1738	1.5498	1.9589	2.398	2.864						
	Expo1	0.0779855	0.248133	0.496401	0.808843	1.17149	1.5702	1.99046	2.41693	2.83288						
	Expo2	0.0934783	0.280255	0.533632	0.836207	1.17734	1.55136	1.95645	2.39374	2.86698						
	NS3	0.0966723	0.28366	0.534463	0.834347	1.17469	1.55035	1.95824	2.39668	2.86489						
	VMI	0.093149	0.27952	0.531326	0.833268	1.1762	1.55407	1.96256	2.39846	2.85922						
^{226}Th	Exp.	0.0722	0.22643	0.4473	0.7219	1.0403	1.3952	1.7815	2.1958	2.6351	3.0971					
	Expo1	0.06488	0.208936	0.423293	0.698886	1.02638	1.39605	1.79756	2.21964	2.64958	3.0721					
	Expo2	0.0724693	0.226514	0.446951	0.72147	1.04026	1.39569	1.7821	2.19563	2.6342	3.0976					
	NS3	0.0735616	0.228103	0.44791	0.721218	1.03904	1.39439	1.78178	2.19681	2.63592	3.0962					
	VMI	0.0730527	0.229274	0.450497	0.723647	1.03992	1.39317	1.7789	2.19369	2.63486	3.1002					
^{228}Th	Exp.	0.057759	0.186823	0.378179	0.6225	0.9118	1.2394	1.5995	1.9881	2.4079						
	Expo1	0.0563664	0.182303	0.370978	0.615362	0.9081	1.24162	1.60753	1.99675	2.39887						
	Expo2	0.0591892	0.188648	0.37903	0.622084	0.91058	1.23823	1.59966	1.99038	2.40676						
	NS3	0.0592171	0.188624	0.378884	0.621846	0.91038	1.23825	1.59998	1.99076	2.40639						
	VMI	0.0584579	0.188576	0.380053	0.623384	0.91107	1.2374	1.59796	1.98927	2.40858						
^{230}Th	Exp.	0.0532	0.1741	0.3566	0.5941	0.8797	1.2078	1.5729	1.9715	2.3978	2.85	3.325	3.812			
	Expo1	0.051	0.166771	0.343282	0.57629	0.86128	1.19335	1.56711	1.97646	2.4142	2.871	3.3358	3.7886			
	Expo2	0.0545	0.1759	0.3578	0.5940	0.8788	1.20665	1.57248	1.97147	2.39907	2.85	3.323	3.812			
	NS3	0.0543	0.175577	0.357465	0.593827	0.878804	1.20685	1.57275	1.97166	2.39906	2.8508	3.323	3.8126			
	VMI	0.05473	0.1782	0.36276	0.60053	0.884728	1.20986	1.57153	1.9662	2.39097	2.8434	3.3216	3.8238			
^{232}Th	Exp.	0.049369	0.16212	0.3332	0.5569	0.827	1.1371	1.4828	1.8586	2.2629	2.6915	3.1442	3.6196	4.1162	4.6318	5.162
	Expo1	0.0469859	0.1537	0.316576	0.531959	0.796087	1.10507	1.45483	1.84111	2.25936	2.7047	3.1717	3.6543	4.1454	4.6366	5.117
	Expo2	0.051554	0.16614	0.3374	0.5595	0.827	1.1353	1.4797	1.8564	2.2618	2.6928	3.1467	3.6216	4.1159	4.6291	5.163
	NS3	0.0521108	0.167346	0.338902	0.560796	0.827747	1.13508	1.47864	1.85473	2.26004	2.6916	3.1468	3.6232	4.1187	4.6314	5.159
	VMI	0.0512957	0.167196	0.340785	0.564827	0.833022	1.14022	1.48228	1.85584	2.25815	2.687	3.1403	3.6166	4.1144	4.6325	5.17
^{234}Th	Exp.	0.04955	0.163	0.3365	0.5648	0.843	1.1602									
	Expo1	0.0500873	0.163789	0.336946	0.564832	0.841828	1.16077									
	Expo2	0.0496468	0.162964	0.336318	0.565063	0.842863	1.16023									
	NS3	0.0493286	0.162752	0.336555	0.565347	0.842452	1.16036									
	VMI	0.0498417	0.16383	0.337332	0.564863	0.841075	1.16119									
^{230}U	Exp.	0.05172	0.1695	0.3471	0.5782	0.8564	1.1757									
	Expo1	0.0514571	0.167754	0.344104	0.57532	0.855654	1.17852									
	Expo2	0.0527099	0.170521	0.347429	0.577644	0.855554	1.17568									
	NS3	0.0525048	0.170207	0.347228	0.577675	0.855746	1.17582									
	VMI	0.0519903	0.169995	0.347769	0.578488	0.855978	1.17505									
^{232}U	Exp.	0.047572	0.15657	0.3226	0.541	0.8058	1.1115	1.4537	1.8281	2.2315	2.6597					
	Expo1	0.0476946	0.155803	0.320368	0.537231	0.80197	1.10981	1.4555	1.8331	2.23572	2.6546					
	Expo2	0.0487539	0.158395	0.32408	0.541061	0.804667	1.11027	1.45326	1.82896	2.23257	2.65898					
	NS3	0.0483989	0.157746	0.323469	0.540791	0.804835	1.11072	1.45364	1.82894	2.23216	2.65907					
	VMI	0.0483552	0.158565	0.325521	0.543394	0.806704	1.1107	1.45139	1.82546	2.23011	2.66301					
^{234}U	Exp.	0.043498	0.143351	0.296071	0.49704	0.7412	1.0238	1.3408	1.6878	2.063	2.4642					
	Expo1	0.0410392	0.134995	0.279609	0.472491	0.711086	0.99263	1.31408	1.67202	2.06249	2.48076					
	Expo2	0.045345	0.146854	0.299674	0.499281	0.741469	1.02234	1.33829	1.68603	2.06253	2.46511					
	NS3	0.0454946	0.14715	0.29998	0.499432	0.741344	1.02191	1.33763	1.68533	2.06207	2.46517					
	VMI	0.0443987	0.145705	0.299409	0.500305	0.743439	1.02448	1.33977	1.68623	2.06129	2.46277					
^{236}U	Exp.	0.045242	0.149476	0.309784	0.52224	0.7823	1.0853	1.4263	1.8009	2.2039	2.6317	3.0812	3.550	4.039		
	Expo1	0.0473149	0.153943	0.315445	0.52748	0.785804	1.08627	1.42481	1.79747	2.20035	2.62965	3.08167	3.55274			
	Expo2	0.0468891	0.152847	0.313739	0.525436	0.783816	1.08476	1.42414	1.79784	2.20173	2.6317	3.0836	3.55328			
	NS3	0.0462335	0.151405	0.311887	0.523718	0.782695	1.08447	1.42465	1.79886	2.20281	2.63235	3.08355	3.55267			
	VMI	0.0473182	0.155263	0.318989	0.532918	0.791754	1.09087	1.42637	1.79498	2.19397	2.62101	3.07413	3.55165			
^{238}U	Exp.	0.044916	0.14838	0.30718	0.5181	0.7759	1.0767	1.4155	1.7884	2.1911	2.6191	3.0681	3.5353	4.0181	4.517	
	Expo1	0.046941	0.152766	0.313113	0.523718	0.780406	1.0791	1.41579	1.78658	2.18764	2.61522	3.06564	3.53531	4.02069	4.51832	
	Expo2	0.0463545	0.151221	0.310636	0.520622	0.77719	1.07633	1.41402	1.78621	2.18883	2.61778	3.06894	3.53814	4.0212	4.51387	
	NS3	0.0457204	0.149839	0.308893	0.519061	0.776266	1.07627	1.41475	1.7874	2.18998	2.61837	3.06865	3.53708	4.02019	4.51473	
	VMI	0.0473447	0.155232	0.318626	0.531797	0.789364	1.08667	1.41981	1.78554	2.18113	2.60429	3.05308	3.52581	4.02106	4.53753	
^{236}Pu	Exp.	0.04463	0.14745	0.30580	0.5157	0.7735	1.0743	1.4136	1.786							
	Expo1	0.045477	0.148921	0.307014	0.516276	0.773025	1.07332	1.41285	1.78687							
	Expo2	0.0450592	0.147994	0.3059	0.51551	0.773093	1.07431	1.414	1.78584							
	NS3	0.0447506	0.147552	0.305695	0.515699	0.773454	1.07438	1.41358	1.786							
	VMI	0.0449603	0.148339	0.306929	0.516682	0.773383	1.07305	1.41211	1.78742							
^{238}Pu	Exp.	0.044076	0.145952	0.30338	0.51358	0.77348	1.0801	1.4291	1.8185	2.2449	2.7057	3.1988	3.7208	4.2652		
	Expo1	0.0452116	0.148251	0.306188	0.516104	0.775095	1.08027	1.42873	1.81761	2.24402	2.70508	3.1979	3.7196	4.2673		
	Expo2	0.0448828	0.147394	0.304836	0.514459	0.773461	1.07898	1.4281	1.81782	2.24508	2.70672	3.1995	3.72014	4.2651		
	NS3	0.0446438	0.146892	0.304237	0.513979	0.773264	1.07912	1.42849	1.81827	2.24537	2.70666	3.1991	3.7197	4.26552		
	VMI	0.0449603	0.148339	0.306929	0.516682	0.773383	1.07305	1.41211	1.78742	2.19624	2.63621	3.10527	3.60161	4.12364		
^{240}Pu	Exp.	0.042824	0.14169	0.294319	0.49752	0.7478	1.0418	1.3756	1.7456	2.152	2.591	3.061	3.56	4.088		
	Expo1	0.0436253	0.142947	0.295022	0.496929	0.745769	1.03866	1.37275	1.74518	2.15313	2.59377	3.0643	3.5619	4.08378		
	Expo2	0.044081	0.14412	0.296843	0.49911	0.7480	1.0403	1.3735	1.7448	2.152	2.592	3.062	3.56	4.087		
	NS3	0.0440134	0.143965	0.296637	0.498903	0.747737	1.04021	1.37351	1.74493	2.15185	2.59179	3.06236	3.56129	4.0864		
	VMI	0.0428118	0.141614	0.294043	0.496966	0.746947	1.04058	1.37466	1.74628	2.15286	2.59208	3.0619	3.56052	4.08632		
^{242}Pu	Exp.	0.04454	0.1473	0.3064	0.5181	0.7786	1.0844	1.4317	1.8167	2.236	2.686	3.163	3.662	4.172		
	Expo1	0.0460464	0.150649	0.310399	0.521881	0.781667	1.08631	1.43235	1.81626	2.23452	2.68351	3.15956	3.65891	4.17768		
	Expo2	0.04536	0.1488	0.3075	0.5183	0.7781	1.0834	1.4308	1.8166	2.237	2.687	3.163	3.660	4.173		
	NS3	0.0448087	0.14769	0.306169	0.517304	0.777768	1.08393	1.43192	1.81777	2.2374	2.68677	3.1619	3.65892	4.17412		
	VMI	0.0456841	0.15072	0.311837	0.524908	0.785642	1.08999	1.43431	1.81541	2.23052	2.67723	3.15345	3.65734	4.18729		
^{244}Pu	Exp.	0.0442	0.155	0.3179	0.535	0.8024	1.1159	1.471	1.8635	2.289	2.742	3.215	3.69	4.149	4.61	
	Expo1	0.0484475	0.158174	0.325141	0.545226	0.814204	1.12772	1.48125	1.87007	2.28915	2.73313	3.19612	3.67153	4.15177	4.62762	
	Expo2	0.0466299	0.153157	0.316661	0.533969	0.801609	1.11574	1.47208	1.86577	2.29123	2.74195	3.21009	3.68599	4.15727	4.60724	
	NS3	0.0439678	0.14713	0.308829	0.526881	0.797669	1.11634	1.47704	1.87325	2.29806	2.74452	3.20586	3.67574	4.14842	4.61882	
	VMI	0.0509134	0.166258	0.339602	0.564046	0.833448	1.14271	1.48768	1.86496	2.27178	2.70581	3.16509	3.64795	4.15295	4.67881	

FIG. 1: Calculated and observed moment of inertia $2\varphi/\hbar^2$ vs. $(\hbar\omega)^2$ for yrast levels of some actinide nuclei.

TABLE II: The fitting parameters of the present model. The fifth column gives the root mean squire deviation, and the last column gives the ratio R_4 ($R_4 = E_{4^+} / E_{2^+}$).

	Exponential model 1			Exponential model 2				NS3			VMI		
Nucleus	$2\varphi_0/\hbar^2$	Δ_0	I_c	$2\varphi_0/\hbar^2$	Δ_0	ν	I_c	A	δ_1	δ_2	C	φ_0	E_{4^+}/E_{2^+}
^{224}Th	251.923	1.23091	28	133.677	0.84768	0.51292	28	26.6293	0.0843929	0.000856876	0.000575482	29.8549	2.896
^{226}Th	222.463	0.910873	28	152.863	0.67945	0.72223	28	37.7112	0.040805	-0.000043929	0.00061324	39.6624	3.136
^{228}Th	229.721	0.79826	28	167.517	0.54773	0.85624	28	48.3623	0.0234766	0.000144337	0.000666218	50.4777	3.235
^{230}Th	191.858	0.507539	28	168.445	0.458102	1.01945	28	53.6257	0.014732	0.000197523	0.000799989	54.1959	3.27
^{232}Th	249.012	0.686136	38	197.041	0.564861	0.84576	38	55.404	0.019448	0.0000481061	0.000694646	57.8597	3.28
^{234}Th	161.175	0.314743	18	281.883	0.860712	7.28661	18	60.5172	0.00112336	0.00067561	0.00106607	59.8047	3.2896
^{232}U	210.512	0.534279	72	183.773	0.426295	1.210	72	60.6966	0.0101185	0.000246927	0.000821665	61.5702	3.291
^{234}U	210.518	0.377382	30	193.329	0.408151	0.9371	30	63.9563	0.0154031	0.000059729	0.000670585	67.0832	3.296
^{236}U	828.231	1.90051	80	18166.4	4.97745	5.7538	80	63.8512	0.00763493	0.000241785	0.000803306	62.9393	3.304
^{238}U	818.497	1.8805	80	176242.	7.23749	8.6853	80	64.6191	0.0072332	0.000241237	0.00076462	62.8802	3.3035
^{236}Pu	215.001	0.505472	30	1174.79	2.19116	10.881	30	66.493	0.00338933	0.000355371	0.00108931	66.4176	3.3038
^{238}Pu	443.544	1.22283	76	6279.86	3.86422	7.0581	76	66.3884	0.00584195	0.000129583	0.00108931	66.4176	3.3113
^{240}Pu	518.975	1.3449	80	292.084	0.782898	1.0114	80	66.8932	0.00935569	0.0000603669	0.00127273	69.835	3.31
^{242}Pu	353.738	1.01702	56	79930.8	6.41919	15.359	56	66.3148	0.00433962	0.000229594	0.00113743	65.3639	3.31
^{244}Pu	258.717	0.756887	38	3067.69	3.18542	12.261	38	68.7927	-0.00515358	0.000538357	0.000747633	58.3512	3.51

It can be seen from Fig. 1 that ^{244}P exhibits a marked backbending at $\hbar\omega \approx 0.25$ MeV while ^{236}U, ^{238}U, ^{230}Th, ^{238}Pu and ^{242}Pu exhibit a smooth upbending at $\hbar\omega > 0.25$ MeV. The rest of U, Th, and Pu isotope are not presented in Fig. 1. because the backbending phenomena in them is not entirely clear due to lack of experimental data for these nuclei. The absence of backbending in these nuclei, in particular ^{240}Pu, can be ascribed to the presence of octupole correlations in them [21]. The observed backbending in ^{244}Pu could be attributed to the sudden alignment of protons out of the $i_{13/2}$ shell and also due to the influence of the Coriolis force [20]. Also, the sudden upbending in the majority of actinide nuclei (at $I \geq 24\hbar$) is caused by the alignment of $J_{15/2}$ neutrons. On the other hand, Fig. 1 clearly illustrates that our calculations concerning the modified exponential model are in fair agreement with the experimental data followed by the NS3 model calculations. The variable moment of inertia predictions have been generally accepted as giving good results only up to the point where backbending occurs.

The additional remarkable aspect of the modified model in comparison with the older model stems from its ability to avoid the restrection on values of R_4, to exeed 3.0, as in Ref. [18], since here the exponent ν is a free parameter.

IV. CONCLUSION

The present work suggests that the modified version of the exponential model predictions give a fairly accurate description of the backbending in actinide nuclei, in support to our previous calculations [17]. Also, the present interpretation goes parallel with the idea that the behavior of the $i_{13/2}$ proton pair and the $J_{15/2}$ neutron pair at high spins play a decisive role for the appearance of backbending phenomena in this mass region. Furthermore, the absence of backbending in some of actinide nuclei supports the presence of the stable octupole deformation which is explained by the increase of barrier height as the frequency increases and the large $B(E1)/B(E2)$ ratios for these nuclei.

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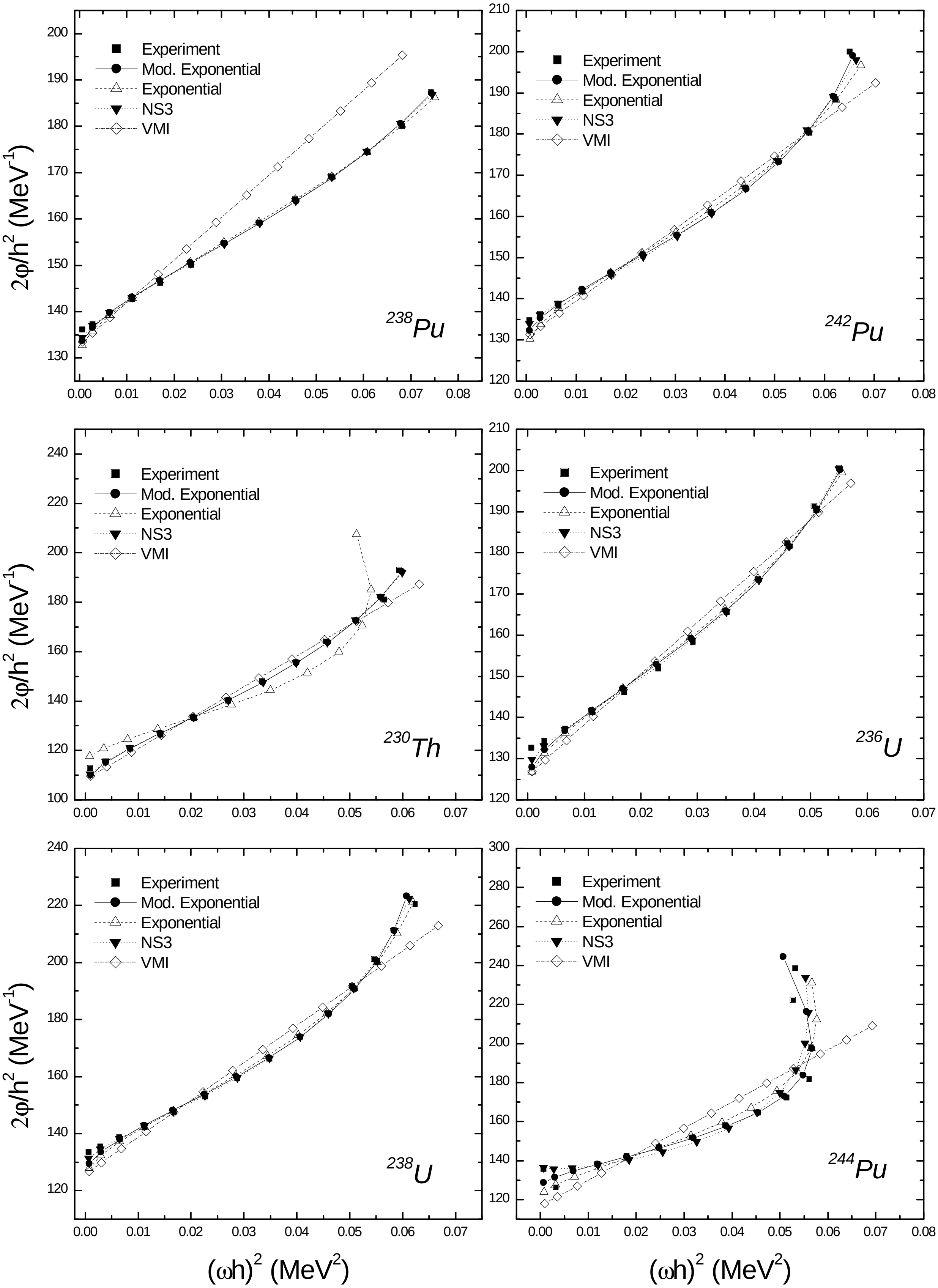

Experiment
Mod. Exponential
Exponential
NS3
VMI
^{238}Pu
^{242}Pu
^{230}Th
^{236}U
^{238}U
^{244}Pu
$2\varphi/\hbar^2$ (MeV^{-1})
$(\omega\hbar)^2$ (MeV^2)